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Superluminal warp drive and dark energy

Pedro F. González-Díaz

Colina de los Chopos, Centro de Física "Miguel A. Catalán", Instituto de Matemáticas y Física Fundamental, Consejo Superior de Investigaciones Científicas, Serrano 121, 28006 Madrid, Spain

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Abstract

In this Letter we consider a warp drive spacetime where the spaceship can only travel faster than light. Restricting to the two-dimensional case, we find that if the warp drive is placed in an accelerating universe the warp bubble size increases in a comoving way to the expansion of the universe in which it is immersed. Also shown is the result that the apparent velocity of the ship steadily increases with time as phantom energy is accreted onto it.
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Considering exotic spacetimes such as wormholes or warp drives can be motivated by pedagogical or conceptual reasons and by the belief that these solutions could play a fundamental role in a consistent theory of quantum gravity to be eventually built up. In this Letter we shall consider how the current accelerated expansion of the universe can modify the kinematic and dynamic characteristics of a restricted kind of two-dimensional warp drives [1,2] which is characterized by having an event horizon and exclusively producing apparent faster than light velocities in the sense that spacetime permits effective superluminal travel even though the speed of light is not locally surpassed [3]. We show that the spacetime of such superluminal warp drives is modified by dark energy by both inflationary and accretion effects. The former effect makes the proper size of the warp drive to increase comovingly with the expansion of the universe while preserving its shape and apparent velocity in such a way that the hypothesis of quantum interest [4] might be violated. The accretion of dark (phantom) energy, on the other hand, would make the apparent velocity of the drive to decrease (increase) so that a big ubiquity of these spacetime constructs should be expected to happen near the big rip singularity.

We shall first briefly review in what follows the properties of the spacetime of a superluminal warp drive. In an Alcubierre warp drive spacetime (t', x, y, z) which is apparently moving along trajectory x_s with velocity $v = dx_s(t')/dt'$, most of the physics that concerns our aims at this Letter is concentrated on the two-dimensional spacetime resulting from setting the coordinates $y = z = 0$, which define the axis about which a cylindrically symmetric space develops. The spacetime of this two-dimensional warp drive contains the entire worldline of the spaceship and, if the apparent velocity of the spaceship is constant, $v = v_0$, then the two-dimensional metric can be written as [2]

$$ds^2 = -A(r)\left[dt' - \frac{v_0(1-f(r))}{A(r)}dr\right]^2 + \frac{dr^2}{A(r)}, \qquad (1)$$

where $r = \sqrt{(x - v_0 t)^2}$ (note that in the past of the spaceship, $x > v_0 t$, $r = x - v_0 t$), $dx = dr + v_0\, dt'$, and

$$A(r) = 1 - v_0^2\bigl(1 - f(r)\bigr)^2. \qquad (2)$$

The function $f(r)$ describes the main kinematic characteristic of the warp drive. In order for the above spacetime to behave like a warp drive, all what is required is that the function $f(r)$ be subjected to the boundary conditions that $f = 1$ at $r = 0$ (the

E-mail address: p.gonzalezdiaz@imaff.cfmac.csic.es.

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location of the spaceship) and $f=0$ at $r=\infty$ [1–3]. A simple choice for $f(r)$ which allows only for constant spaceship apparent velocities $v_0 \geqslant 1$ while preserving the horizon structure of the warp drive (see Eqs. (4) and (5) below) and satisfies the above boundary conditions is

$$f(r)=1-\tanh(\sigma r), \tag{3}$$

with σ any positive constant. Because it is simple enough but still retains all the warp drive properties in the most interesting case that $v_0 \geqslant 1$, this is the choice for $f(r)$ that we shall consider throughout this Letter.

Metric (1) can finally be given a comoving, manifestly static form if we introduce the proper time $dt = dt' - v_0(1-f(r))\,dr/A(r)$. In this case, we in fact derive a two-dimensional spacetime with line element [5]

$$ds^2 = -A(r)\,dt^2 + \frac{dr^2}{A(r)}. \tag{4}$$

This metric shows an apparent singularity at the event horizon that occurs whenever $A(r_0)=0$ if and only if $v_0 \geqslant 1$. For a function $f(r)$ as given by Eq. (4) such an event horizon takes place at

$$r_0 = \frac{\operatorname{arccotanh}(v_0)}{\sigma}, \tag{5}$$

which is defined only for $v_0 \geqslant 1$. We note that if the spaceship moves at the speed of light then the event horizon is shifted to infinity (i.e. it does not exist), and if it does at unboundedly high speeds then the event horizon tends to concentrate onto the very position of the spaceship. Of course, the same horizon structure also appears if we take the Alcubierre ansatz for $f(r)$ [1]. In fact, in this case an event horizon will only occur when $v_0 > 1$ at

$$r_0 = \frac{1}{\sigma}\operatorname{arccoth}\bigl[v_0 - (v_0-1)\tanh^2(\sigma R)\bigr] \tag{6}$$

where R is another arbitrary constant. It is also worth noticing that the above warp drive can also be converted into a time machine for the crew of the spaceship by making the interior of the bubble multiply connected, that is satisfying the usual identification properties of the three-dimensional Misner space [5].

We will now concentrate on investigating the kinematic effects on the shape and size of a two-dimensional warp drive induced by the current accelerated expansion of the universe driven by a dark or phantom energy fluid [6]. We shall follow the procedure put forwards by Roman for inflating wormholes [7], later extended by the author to the cases of comovingly expanding wormholes and ringholes [8]. Following then these works, we shall now generalize the static metric (4) to a time-dependent background metric describing the time-evolution of an initially static two-dimensional warp drive with metric (4) immersed in an accelerating universe. Assuming then that the accelerating expansion is driven by a quintessential dark energy represented by a scalar field with equation of state $p = w\rho$ for which the scale factor is given by $a(t)$, we shall insert a dimensionless factor proportional to the square of the scale factor [8],

$$g(t)^2 = \left(1+\frac{3(1+w)\sqrt{8\pi GR/3}(t-t_0)}{2a_0^{3(1+w)/2}}\right)^{2/[3(1+w)]}, \tag{7}$$

with a_0 the scale factor at $t=t_0$ and R a real and positive constant, in the one-dimensional spatial part of the two-dimensional metric (4). In the most general case in which we add to the dark energy fluid a positive cosmological constant Λ, such a factor would read instead [8]

$$g(t)^2 = \left(e^{\frac{3(1+w)\sqrt{\lambda}(t-t_0)}{2}} - Ce^{-\frac{3(1+w)\sqrt{\lambda}(t-t_0)}{2}}\right)^{\frac{2}{e(1+w)}}, \tag{8}$$

where $\lambda = \Lambda/3$ and

$$C = \frac{\sqrt{\lambda + 8\pi GR/3a_0^{-3(1+w)}} - \sqrt{\lambda}}{\sqrt{\lambda + 8\pi GR/3a_0^{-3(1+w)}} + \sqrt{\lambda}}. \tag{9}$$

We have then for the time-dependent warp drive metric in an accelerating universe

$$\begin{aligned} ds^2 &= -A(r)\,dt^2 + g(t)^2\frac{dr^2}{A(r)} \\ &= -\bigl(1-v_0^2\tanh^2(\sigma r)\bigr) + \frac{g(t)^2\,dr^2}{(1-v_0^2\tanh^2(\sigma r))}. \end{aligned} \tag{10}$$

We shall choose a coordinate system which is comoving with the warp drive geometry and consider an originally nearly Planck-sized warp drive in order to ensure full quantum stability of our construct. Then, for a radial proper length through the warp drive between any two points A and B we should have for $A=0$ and $B=r_0$,

$$\begin{aligned} d(t) &= \pm g(t)\int_0^{r_0}\frac{dr'}{\sqrt{1-v_0^2\tanh^2(\sigma r')}} \\ &= \pm\frac{g(t)}{\sigma}F\bigl[\arcsin\bigl(\tanh(\sigma r)\bigr), v_0\bigr], \end{aligned} \tag{11}$$

with the function F being the elliptic integral of the first kind. It follows that for fixed initial values r_0 and v_0 the size of the warp drive one-dimensional bubble increases with time t at exactly the same rate as that of the current universal expansion, in such a way that it always holds

$$\tanh(\sigma r_0) = \operatorname{sn}\left(\frac{\sigma\, d(t)}{g(t)}\right),$$

with sn the hyperbolic sinus function.

We shall now consider the effects of accelerating expansion on the shape of the warp drive. This will be done using the embedding of a $t=\text{const}$ slice of the warp drive spacetime in a flat two-dimensional Euclidean space described by the metric

$$ds^2 = d\bar{z}^2 + d\bar{r}^2. \tag{12}$$

Since the metric on the chosen slice should be given by

$$ds^2 = g(t)^2\,dr^2/A(r), \tag{13}$$

we get

$$d\bar{r}^2 = g(t)^2\,dr^2, \quad \bar{r} = g(t)r. \tag{14}$$

It follows that, relative to the $(\bar{z}, \bar{r})$ coordinates, the form of the warp drive metric will be preserved provided that the metric on the embedded slice had the form

$$ds^2 = \frac{d\bar{r}^2}{1 - v_0^2 \tanh^2(\bar{\sigma}\bar{r})}, \tag{15}$$

in which $v_0^2 \tanh^2(\bar{\sigma}\bar{r})$ has to have a minimum at some $\bar{r}_0 = \bar{\sigma}^{-1}\mathrm{arccotanh}(v_0)$. By using then transformations (14), it can be readily seen that Eqs. (13) and (15) can be re-written into each other if we also take $\bar{\sigma} = \sigma/g(t)$. It follows finally that the shape of the warp drive is preserved during the current accelerating expansion of the universe.

Combining the above two results one can conclude that whereas the amount of overcompensation of negative energy by positive energy in the warp drive is kept invariant, the space-like separation, and hence the time interval between the locations of such energies, steadily increase with time for a causal distant observer, as the universe expands in an accelerated fashion. This implication manifestly violates the quantum interest conjecture of Ford and Roman [4] according to which any positive energy pulse must overcompensate the negative energy pulse by an amount which is a monotonically increasing function of the pulse separation. We have thus found what, to our knowledge, could be the first explicit counter example of such a conjecture.

We will argue in what follows that a warp drive immersed in dark energy should accrete a steady flow of such energy and that therefore the gravitational potential created by that geometrical construct should be non-vanishing. Actually, it could be objected that there is no dynamic model for a warp drive which is just a designer spacetime. However, it is also true that negative energy is required on the sides of the bubble. It is this negative energy which will provide the spacetime construct with a dynamic content that is unspecified unless by the fact that the larger the apparent spaceship velocity the more negative energy is required to keep up the spaceship.

Even though we have not a proper dynamic description for two-dimensional warp drives, the spacetime structure described by metric (4) appears to be static de Sitter alike and therefore, for a metric ansatz $ds^2 = -e^{\nu}\,dt^2 + e^{\mu}\,dr^2$, the two main 00 and 11 components of the Einstein equations,

$$\frac{\lambda'}{r}e^{-\lambda} + \frac{1}{r^2}\left(e^{-\lambda} - 1\right) = T_{00}, \tag{16}$$

$$\frac{\nu'}{r}e^{-\lambda} - \frac{1}{r^2}\left(e^{-\lambda} - 1\right) = T_{11}, \tag{17}$$

would be associated with components of the stress-energy tensor that satisfy $T_{00} = -T_{11}$. In the two-dimensional case the time component of the stress tensor, T_{00}, can then be chosen to be [3]

$$T_{00} = -T_{11} = \rho = \frac{v_0^2 \tanh^2(\sigma r)}{r^2}\left(\frac{4\sigma r}{\sinh(2\sigma r)} - 1\right), \tag{18}$$

with ρ the energy density. It is worth noting that although near the position of the spaceship one can have that the energy density $\rho \simeq v_0^2\sigma^2 > 0$, ρ will steadily decrease with r to vanish at $r = \sinh(2\sigma r)/(4\sigma)$, becoming increasingly negative thereafter until it reaches a new extremal value, to finally again vanish as $r \to \infty$, such as one should expect for a warp drive spacetime [1,2]. It follows that the gravitational potential,

$$U = -v_0^2 \tanh^2(\sigma r)\left(\frac{4\sigma r}{\sinh(2\sigma r)} - 1\right), \tag{19}$$

is generally nonzero and therefore a two-dimensional warp drive in an accelerating universe should accrete a steady flux of the background dark or phantom energy whatever the precise dynamics it could obey.

It follows—and this is our content here—that the apparent velocity can be hypothesized to be proportional to the amount of negative energy. The resulting accretion effect would be thus added to the kinematic effect studied above and in principle compete with the thermal emission considered in Ref. [3]. In order to investigate the accretion of dark energy by our two-dimensional warp drive, we shall follow the time-dependent procedure based on the integration of the conservation laws [9] that can be prescribed in the considered scenario, allowing for the warp drive metric to be no longer static [10] as v_0 is considered to be time-dependent. In the present case, the conservation law for the energy–momentum tensor, $T^{\nu}_{\mu;\nu} = 0$ can be integrated to yield

$$u(p+\rho)\frac{\sqrt{1 - v_0^2(1 - f(r))^2 + u^2}}{1 - v_0^2(1 - f(r))^2} \times \frac{\mathrm{arccotanh}^2(v_0)}{\sigma^2} e^{\int \beta(r,p,\rho)\,dr} = C, \tag{20}$$

where $u \equiv u^r = dr/ds$ is the r-component of the four-velocity, with $u_\mu u^\mu = -1$, p and ρ are the pressure and energy density, respectively, of the dark energy perfect fluid with stress-tensor $T_{\mu\nu} = (p+\rho)u_\mu u_\nu + p g_{\mu\nu}$, the factor $r_0^{-2} = \frac{\mathrm{arccotanh}^2(v_0)}{\sigma^2}$ has been introduced to make the constant C dimensionless, and

$$\beta(r,p,\rho) = \frac{\partial_0\left[(p+\rho)\frac{(1-v_0^2(1-f(r))^2+u^2)(1-v_0^2(1-f(r))^2+p)}{[1-v_0^2(1-f(r))^2]^2}\right]}{(p+\rho)u(1 - v_0^2(1 - f(r))^2 + u^2)^{1/2}} + \frac{v_0\dot{v}_0(1 - f(r))^2(1 - v_0^2(1 - f(r))^2 + 2u^2)}{(1 - v_0^2(1 - f(r))^2)^2 u(1 - v_0^2(1 - f(r))^2 + u^2)^{1/2}}, \tag{21}$$

with the overhead dot meaning time derivative. The integration of the conservation law for the projection onto the four-velocity of the stress-energy tensor, $u^\mu \rho_{,\mu} + (p+\rho)U^{\mu}_{;\mu} = 0$, would in turn lead to the expression

$$u\frac{\sigma^2}{\mathrm{arccotanh}^2(v_0)} e^{\int \frac{d\rho}{p+\rho}} e^{\int \alpha(r,p,\rho)\,dr} = A, \tag{22}$$

in which we have again introduced a factor r_0^{-2} to make the integration constant A to have the appropriate physical dimensions, and

$$\alpha(r,p,\rho) = \frac{(1 - v_0^2(1 - f(r))^2 + u^2)^{1/2}}{u(1 - v_0^2(1 - f(r))^2)}\frac{\partial_0\rho}{p+\rho} + \frac{\partial_0 u^0}{u}. \tag{23}$$

Now, from Eqs. (20) and (22) we can derive the following useful relation

$$(p+\rho)\frac{(1-v_0^2(1-f(r))^2+u^2)^{1/2}}{1-v_0^2(1-f(r))^2} \times \exp\left[\int dr\left(\beta(r,p,\rho)-\alpha(r,p,\rho)-\frac{d\rho}{p+\rho}\right)\right] = \frac{C}{A} = B, \tag{24}$$

with B independent of r. Let us then introduce the rate equation for the variation of the spaceship velocity which we take to be

$$\dot{v} = -\Phi T_0^r = -\Phi AB\left[1-v_0^2\left(1-f(r)\right)^2\right]\frac{\operatorname{arccotanh}^2(v_0)}{\sigma^2} \times \exp\left(-\int dr\,\beta(r,p,\rho)\right), \tag{25}$$

where Φ is a numerical proportionality constant encapsulating the so far unknown way in which apparent velocity and negative mass are linearly related to each other in the present case, and we have used Eqs. (20) and (24). Other possible relations between v and the amount of negative energy would lead to different expressions for $\dot{v}$. In what follows we shall restrict ourselves to the linear dependence between v and negative mass implied by Eq. (25), leaving other possible relations to be considered elsewhere.

We shall discuss in what follows the limits in the integral of the exponent of the above expression. If we take B to be a function of t then, in the limit $r\to\infty$, we have $B = p(\rho_\infty(t))+\rho_\infty(t)$. The integration limits would then generally correspond to $\int_r^\infty dr\,\beta(r,p,\rho)$. It follows therefore that, allowing v_0 to become $v(t)$, for an asymptotic observer the previous expression reduces asymptotically ($r\to\infty$) finally to

$$\dot{v} = -\Phi A\sigma^{-2}(p+\rho)\left(1-v^2\right)\operatorname{arccotanh}^2(v). \tag{26}$$

This equation can now be integrated to finally yield

$$v = \operatorname{cotanh}\left[\frac{\operatorname{arccotanh}(v_0)}{1+\frac{\Phi A\operatorname{arccotanh}(v_0)(1+w)(t-t_0)}{\sigma^2 a(t)^{3(1+w)/2}}}\right], \tag{27}$$

where we have used the equation of state $\rho = p/w = \rho_0 a(t)^{-3(1+w)}$, with $a(t)$ the scale factor of the accelerating universe [6], that is for the simplest quintessence field case,

$$a(t) = a_0\left(1+\frac{3}{2}(1+w)\sqrt{C}(t-t_0)\right)^{2/[3(1+w)]},$$

with $C = 8\pi GR/3$, we see that for dark energy with $w>-1$ the velocity of the spaceship for an asymptotic observer will steadily decreases with time, tending to the constant value

$$v_\infty = \operatorname{cotanh}\left(\frac{\operatorname{arccotanh}(v_0)}{1+\frac{2\Phi A\operatorname{arccotanh}(v_0)}{3\sigma^2\sqrt{C}a_0^{3(1+w)}}}\operatorname{arccotanh}^2(v)\right),$$

as $t\to\infty$. In the case of phantom energy, i.e. for $w<-1$ [11], the velocity of the spaceship for an asymptotic observer will progressively increases and tends to infinite as one approaches the big rip singularity at $t_{bp} = t_0 + \frac{2}{3(|w|-1)\sqrt{C}}$. This big ubiquity making the spaceship to be present everywhere in an infinite space would in principle make all existing infinitely spread out elementary objects in the universe at the singularity to be accessible to the observer in the warp drive, even at the big rip singularity. However, such an ubiquitous behaviour of the ship would take place when its temperature diverges so that it consistently becomes a part of that singularity.

The model for accretion of dark and phantom energy that we have just considered would also suffer from the kind of difficulties pointed out by Faraoni [12] for the case of wormholes, namely, that (i) it cannot be adjusted to satisfy the Einstein equations as the used conservation laws would only strictly correspond to vacuum solutions, and (ii) the velocity of the phantom fluid is proportional to $a^{3(1+w)/2}$ and hence the phantom energy accretion process is being stopped as one approaches the big rip singularity. However, the first of these difficulties is rather formal—if not misled—as our accretion process is referred to an asymptotic observer at $r\to\infty$ for which the components T_{00} and T_{11} both strictly vanish, rendering the whole mechanism to be fully equivalent to a vacuum process. Difficulty (ii) is also meaningless because what matters for accretion is by no means the velocity of the fluid but clearly its flow which is given by the amount of fluid energy which is accreted per unit surface per unit time and this is instead given by $\sim a^{-3(1+w)/2}$ and steadily increases as one approaches the big rip singularity. Thus, such as it happens for wormholes, the kind of criticisms raised by Faraoni would neither apply to two-dimensional warp drives.

This Letter deals with some physical aspects of a special kind of two-dimensional Alcubierre warp drive which always describes spaceships moving at the speed of light or at superluminal apparent velocities. After reviewing the spacetime of such constructs, we considered the kinematic effects that an accelerating universe filled with dark energy have on a warp drive. It was concluded that the size of the warp drive increases in a comoving way to as the accelerating Universe does, without changing its apparent velocity. This result actually leads to a violation of the so-called quantum interest conjecture, according to which any positive energy pulse that separates from a negative energy pulse must overcompensate this by a quantity which increases with the increasing distance between the two pulses. Finally, we have investigated the accretion of dark and phantom energy onto a warp drive with time-dependent apparent velocity. We obtained that the accretion of dark energy producing acceleration slower than de Sitter space makes the warp drive velocity to steadily decreases with time until reaching a constant minimum value asymptotically. In case that the warp drive accretes phantom energy its velocity would increase with time, tending to infinite as the phantom universe approaches the big rip singularity. That implies that the warp drive tends to become maximally ubiquitous as the singularity is approached.

Even so, there is a point which deserves further consideration. It is that there are two at first sight different mechanisms that induce a decrease of the positive/negative energy balance in the warp drive. On the one hand, the thermal process by which the warp drive radiates at a given finite temperature [3]. In this case, one would expect the apparent velocity and the temperature of the spaceship to both increase as a consequence from the loss of positive energy. On the other hand, accretion of phantom energy leads to the same overall effect. The above two processes can be actually seen to be indistinguishable for warp drives if one wants to respect the laws of quantum theory. In fact, if positive energy must always overcompensate negative energy, then a steady internal process of creation of positive energy should a way or another be added to the accretion of phantom energy by the warp drive. Possibly second law of thermodynamics ought to require the created positive energy to be in the form of thermal particles, so unifying the two processes into a unique, single mechanism.

We finally remark that, since most of the warp drive physics depending on the spaceship worldline is concentrated on time and the space direction along which it apparently moves, the conclusions drawn in this Letter for two dimensions, concerning the effects of the accelerating expansion of the universe and accretion of dark and phantom energy, ought to be still valid in the corresponding three-dimensional version. A caveat is nevertheless in order. The energy density measured by Eulerian observers in the three-dimensional warp drive becomes zero in the used two-dimensional limit and it is unclear how such an energy density may affect the accretion process, though one would expect that it slightly slowed that process down, at least in its earliest stages.

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